Balcatta Senior High School
Building the Future

Mathematical Methods

Polynomial Functions Practice Test

Name ANNE SIRS.

Non calc	/ 21
Calc	/ 30
Total	/ 51
Percentage	

SECTION ONE: RESOURCE FREE

TOTAL: 21 marks

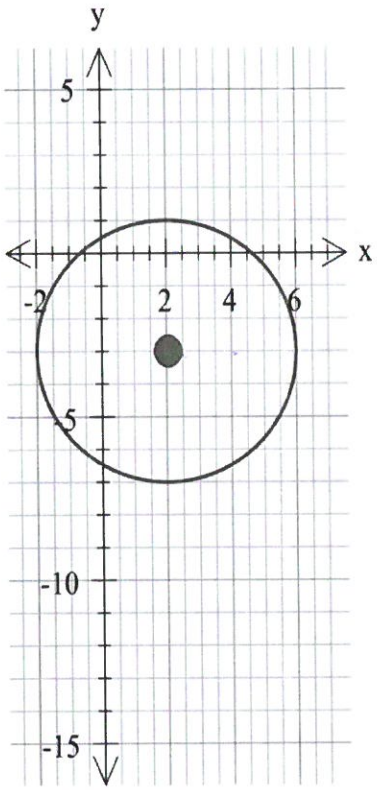
EQUIPMENT: pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet

WORKING TIME: 21 minutes

Show all of your working clearly. *Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.*

Question 1 **[2,2,4 = 8 Marks]**

a. Find the equation of the circle drawn, you do not need to expand or simplify your answer.



Centre at $(2, -3)$

Radius of 4

$$(x-2)^2 + (y-(-3))^2 = 4^2$$

$$(x-2)^2 + (y+3)^2 = 16$$

b. Give the domain and the range of this circle.

domain $\{x: -2 \leq x \leq 6\}$

range $\{y: -7 \leq y \leq 1\}$

c. Find the centre and radius of the circle defined by:

$$x^2 + y^2 - 8x + 6y - 24 = 0$$

Use completing the square:

$$(x^2 - 8x) + (y^2 + 6y) - 24 = 0$$

$$((x-4)^2 - 16) + ((y+3)^2 - 9) - 24 = 0$$

$$(x-4)^2 + (y+3)^2 - 49 = 0$$

$$(x-4)^2 + (y+3)^2 = 7^2$$

Centre at $(4, -3)$ radius $= 7$

Question 2 **[1,1,3,2 = 7 Marks]**

For the graph of $y = x^3 - x^2 - 4x + 4$, determine:

a. the coordinates of the point where the curve cuts the y-axis,

$(0,4)$

b. If $y = f(x)$, what is $f(2)$?

$f(2) = 2^3 - 2^2 - 4(2) + 4$

$= 8 - 4 - 8 + 4 = 0$

c. the coordinates of the points where the line cuts the x-axis

$x - 2$ must therefore be a factor

$$x-2 \overline{)\, x^3 - x^2 - 4x + 4}$$

$x^2 + x - 2$

$x^3 - 2x^2$

$x^2 - 4x + 4$

$x^2 - 2x$

$-2x + 4$

$-2x + 4$

0

$\frac{x^3}{x} = x^2$

$x^2(x-2) = x^3 - 2x^2$

$\frac{x^2}{x} = x$

$x(x-2) = x^2 - 2x$

$\frac{-2x}{x} = -2$

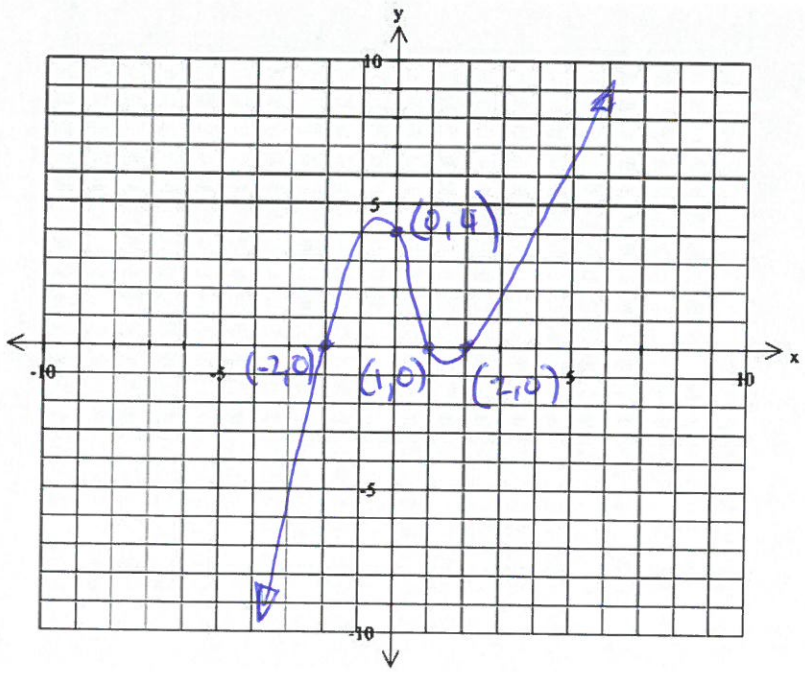
$f(x) = (x-2)(x^2+x-2)$

By inspection:

$f(x) = (x-2)(x+2)(x-1)$

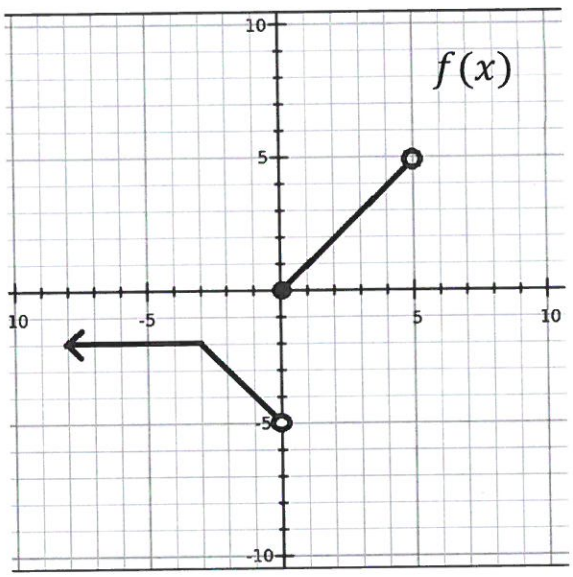
$\therefore (2,0), (-2,0), (1,0)$

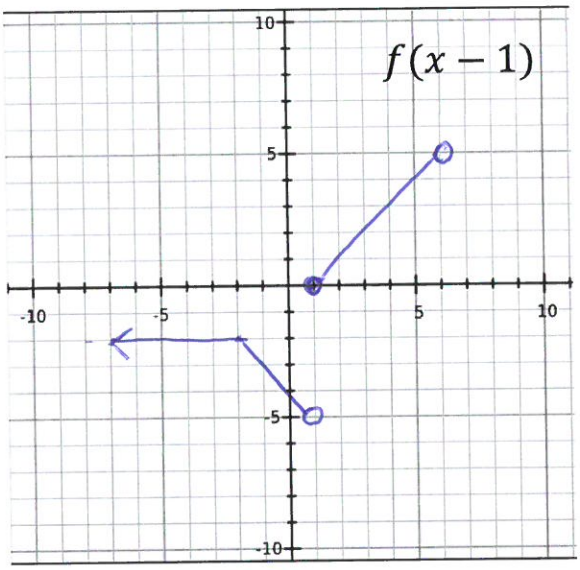
d. Draw a sketch of this graph

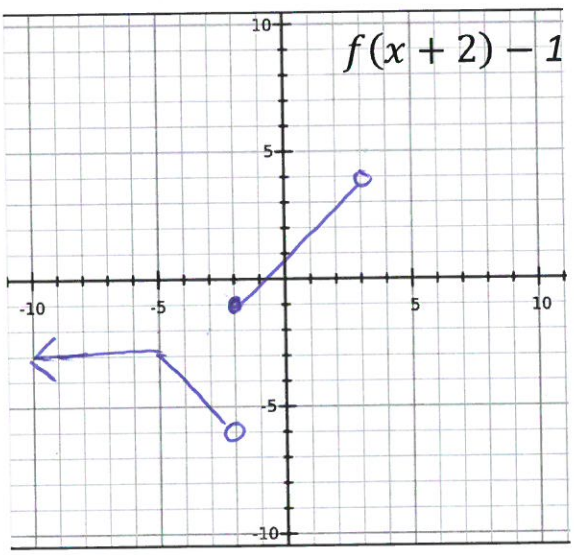


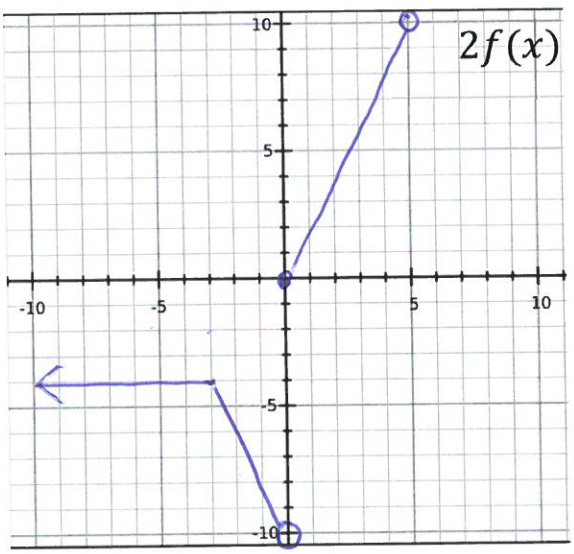
Question 3 **[2,2,2 = 6 Marks]**

The following is the graph of $f(x)$. On the graphs below, sketch the new graph with the transformations applied









Question 4 **[3,2 = 5 Marks]**

a. Describe the transformation that takes place when $y = x^3$ is changed to,

$$y = 2(3x - 4)^3 + 5$$

Vertically dilated by factor of 2.
Horizontally dilated by factor of $\frac{1}{3}$.
Horizontally translated 4 units to right
Vertically translated 5 units upwards.

b. The original graph contains the point $(5, 125)$, where will this point be transformed to in the graph of

$$y = 2(3x - 4)^3 + 5$$

$(5, 125)$
VD $(5, 250)$
HD $\left(\frac{5}{3}, 250\right)$
HT $\left(\frac{5}{3} + 4, 250\right)$
VT $\left(\frac{5}{3} + 4, 250 + 5\right)$

$\therefore \left(\frac{17}{3}, 255\right)$

END OF SECTION 1

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SECTION TWO: RESOURCE ALLOWED

TOTAL:	30 marks
EQUIPMENT:	pens, pencils, pencil sharpener, highlighter, eraser, ruler, SCSA formula sheet, scientific &/ or CAS calculator, 1 A4 page of notes
WORKING TIME:	30 minutes

Show all of your working clearly. *Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks.*

Question 1 **[3 Marks]**

Determine the equation of the graph below.

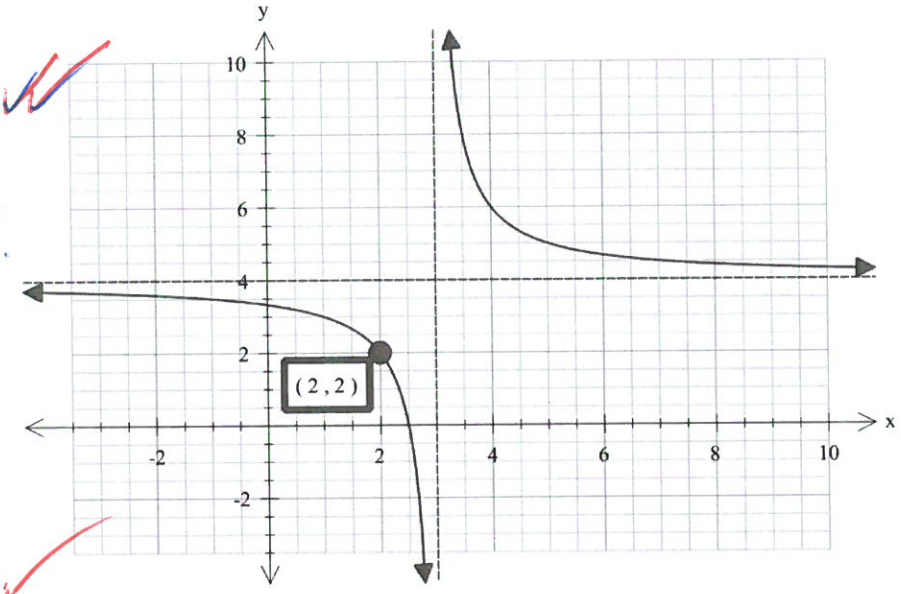
$y = \frac{a}{x-3} + 4$ ✓

$2 = \frac{a}{2-3} + 4$

$-2 = \frac{a}{-1}$

$a = 2.$

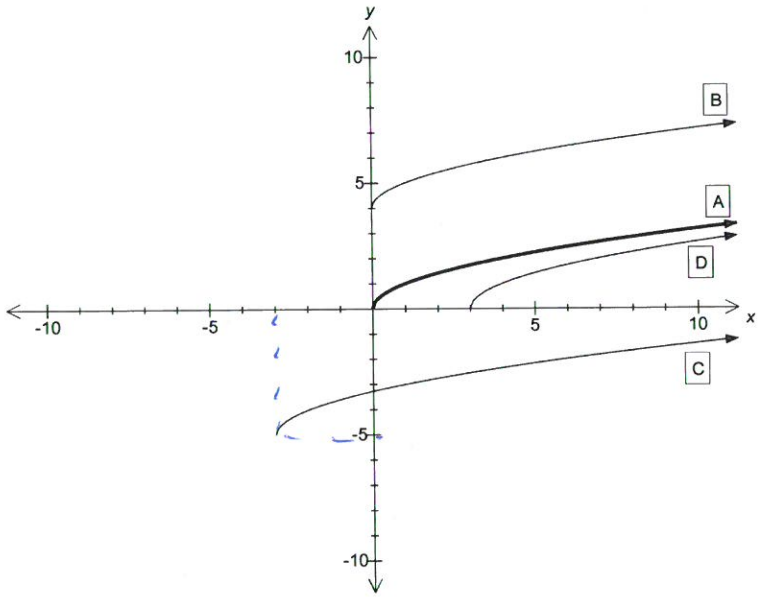
$y = \frac{2}{x-3} + 4$ ✓



Question 2 **[3 Marks]**

Graph A shown in bold below has equation $y = \sqrt{x}$.

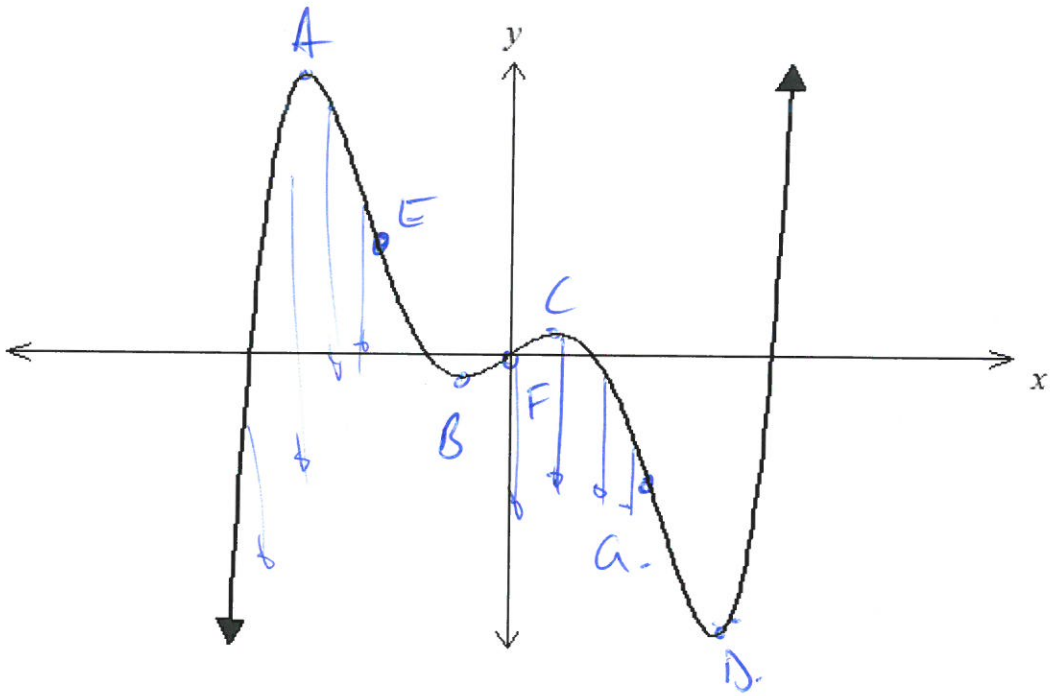
Graphs B, C and D are all translations (no dilations) of graph A. Write down the equations of B, C and D



Graph B	$y = \sqrt{x} + 4$
Graph C	$y = \sqrt{x+3} - 5$
Graph D	$y = \sqrt{x-3}$

Question 3 **[4,2 = 6 Marks]**

The graph below is that of the curve $y = x^5 - 6x^3 + 3x$



Use your calculator to help find:

a. Find the co-ordinates of each of the turning points.

A (-1.85, 10.77) ✓
B (-0.42, -0.83) ✓
C (0.42, 0.83) ✓
D (1.85, -10.77) ✓

b. Find the intervals where the curve is concave downwards.

Points of inflection at: E(-1.342, 6.118)
F(0, 0).
G.(-1.342, -6.118)

Concave downwards between:

$-\infty \leq x \leq -1.342$. ✓

and

$0 \leq x \leq 1.342$. ✓

Question 4 **[1,4,1,2 = 8 Marks]**

The depth of water in a flask in a science experiment was measured for an eight hour period and followed the rule:

$$d = 0.4t^3 - 5t^2 + 16t + 2$$

Where t was the number of hours since the experiment began and d the depth of water in centimetres.

a. What was the depth of water when the experiment began?

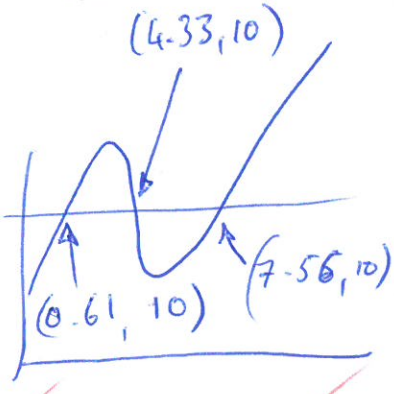
2cm. ✓

b. In total for how many hours and minutes was the depth 10cm or more during the eight hour period?

graph $y = 0.4t^3 - 5t^2 + 16t + 2$.

and $y = 10$

Find intersections.



~~$0 \leq t \leq 0.61$~~ $0.61 \leq t \leq 4.33$ ✓ $= 3.72$ hrs

$7.56 \leq t \leq 8$ ✓ $= 0.44$ hrs.

$= 4.16$

Total = 4hr, 10mins. ✓

c. What was the minimum depth of water during the first eight hours?

2cm. ✓

d. At what time, correct to nearest minute, was the depth a maximum during the first eight hours?

2.16 hr in. ✓

2 hr, 10mins. ✓

Question 5 **[1,3,1,2,3 = 10 Marks]**

A recent study into the annual profitability, $P(B)$, in dollars, of individual hospitals owned by a consortium found that it varied with the number of beds, B, in each hospital.

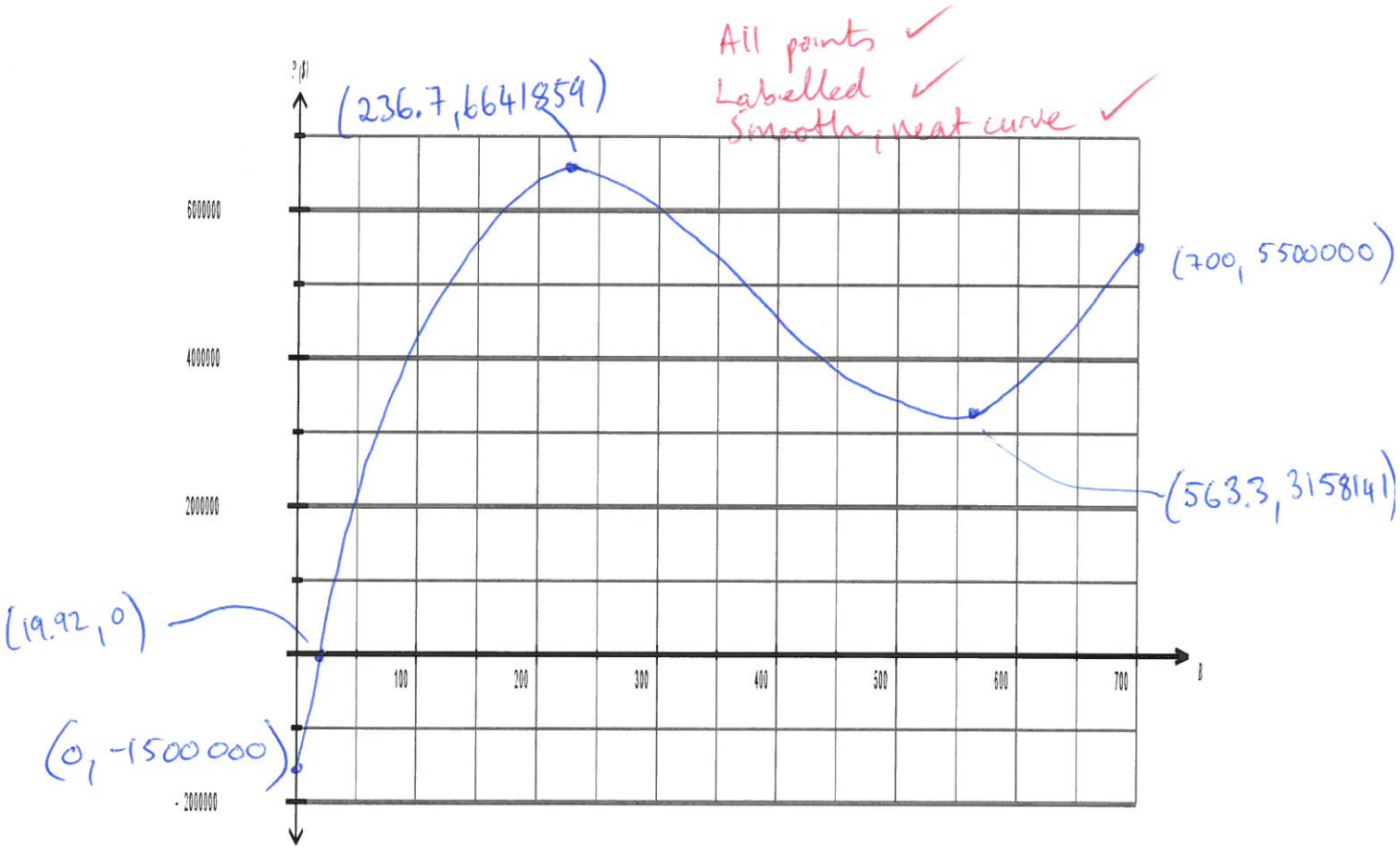
Using the model:

$$P(B) = 0.2B^3 - 240B^2 + 80000B - 1500000$$

a. What annual profit is expected if a hospital has 50 beds?

$1 925 000 ✓

b. Graph $P(B)$ on the axes below showing all intercepts, turning points and P(700).



c. What is the fewest number of beds a hospital needs to make a profit rather than a loss?

20 ✓

d. A new hospital is being planned for the consortium with between 100 and 400 beds. What number of beds would you recommend and why?

237 beds. Maximise profit (turning point here)

e. The consortium are considering closing two smaller hospitals, with 250 and 310 beds, and replacing them with a single hospital to accommodate all their patients. Discuss whether this would be an effective strategy to increase profitability.

Both separate [P(250) = $6625000 ; P(310) = $6194200] + = $12 819 200

Together [P(560) = $3159 200]

Terrible strategy. Over $9 million profitability loss

END OF SECTION 2